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HIGH EFFICIENCY GEARS

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Abstract: *The paper presents an original method for determining gear efficiency, gearing forces, velocities and powers. It analyzes the way in which certain parameters affect gear efficiency. Furthermore, an original method for determining geared transmissions efficiency as a function of the contact ratio is concisely presented. With the presented relations, one can make a dynamic synthesis of geared transmissions with the aim of increasing gearing mechanisms efficiency.*

Key Words: *Gear, Geared Transmission, Contact Ratio, Dynamic Synthesis, Gear Efficiency*

1. INTRODUCTION

Today gears are present in all possible fields. Their key advantage lies in their providing for a very high working efficiency. Additionally, gears can transmit large loads [3, 13]. Regardless of their size, gears must be synthesized carefully considering the specific conditions. This paper intends to present the main conditions that must be met for a correct synthesis of the gear [8-11].

The pinnacle of using sprocket mechanisms is to be sought in ancient Egypt at least some thousand years before Christ. For the first time in human history, transmission wheels "spurred" to irrigate crops were used and so were worm gears for cotton processing [1-2].

In the year of 230 B.C., in the city of Alexandria in Egypt, multi-lever wheels with gear rack were used. Such gears were constructed and used from primeval times, above all for lifting heavy anchors of vessels and for winding catapult arms on the battlefields. They were later introduced in the vehicles with wind and water in order to reduce or enlarge the pump originating from windmills or water (see Fig. 1).

The Antikythera Mechanism is the name given to an astronomical calculating device, measuring about 32 by 16 by 10 cm, which was discovered in 1900 in a sunken ship just off the coast of Antikythera, an island between Crete and the Greek mainland. Several kinds of evidence point unquestionably to around 80 B.C. as the date of the shipwreck. The device, made of bronze gears fitted in a wooden case, was crushed in the wreck, and parts of the faces

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were lost, "the rest then being coated with a hard calcareous deposit at the same time as the metal corroded away to a thin core coated with hard metallic salts preserving much of the former shape of the bronze" during almost 2000 years of its underwater existence (Fig. 2).

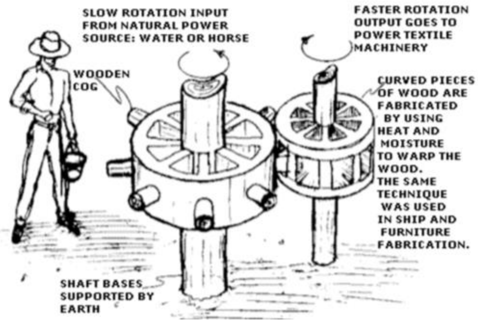

Fig. 1 Transmissions wheeled "spurred" to irrigate crops and worm gears used for cotton processing

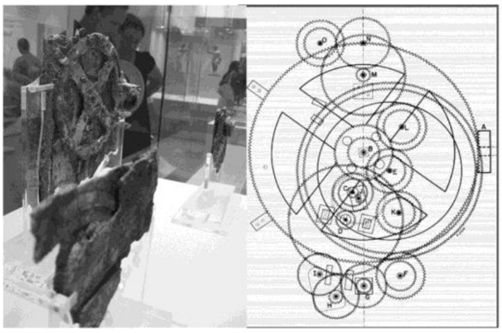

Fig. 2 The "Antikythera" mechanism – an astronomical calculating device

Modern adventure began with the spur gear wheel spurred created by Leonardo da Vinci, in the 15th century. He founded the new kinematics and dynamics stating *inter alia* the principle of superposition of independent movements (Fig. 3).

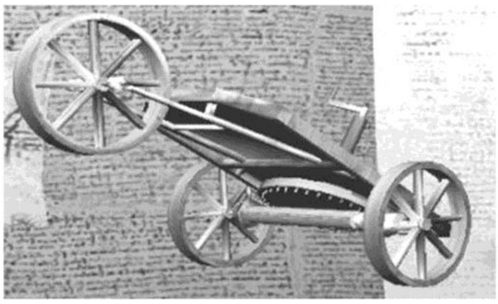

Fig. 3 The spur gear wheel spurred created by Leonardo da Vinci, 15th century

Benz had engine with transmissions sprocket gearing and gear chain (patented in 1882, Fig. 4), [4-6] but the first gear patent (the drawings of the first gear transmission patented) and gearing wheels with chain were made in 1870 by the British Starley & Hillman [12].

Fig. 4 The Benz patent

It is in 1912, in Cleveland (USA), that the production of industrial specialized wheels and gears (cylindrical, worm, conical, with straight teeth, inclined or curved; see Fig. 5) started.

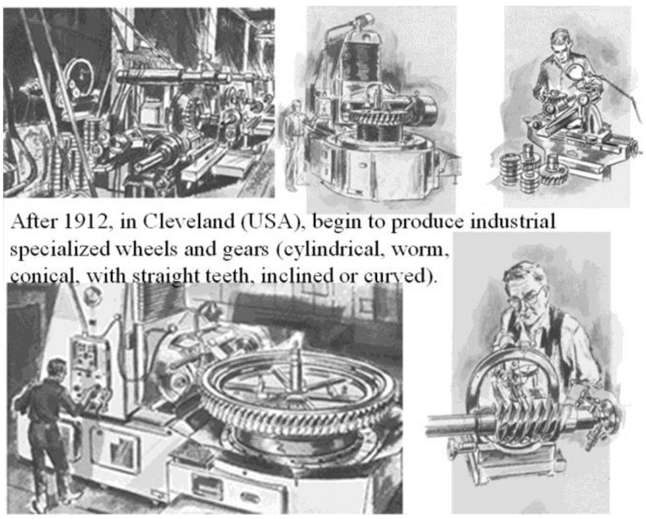

Fig. 5 In Cleveland, the production of industrial specialized wheels started in 1912

Today, the gears are present everywhere in the world of mechanics, such as the car industry, electronics and electro-technique equipments, power industry, etc. (Fig. 6).

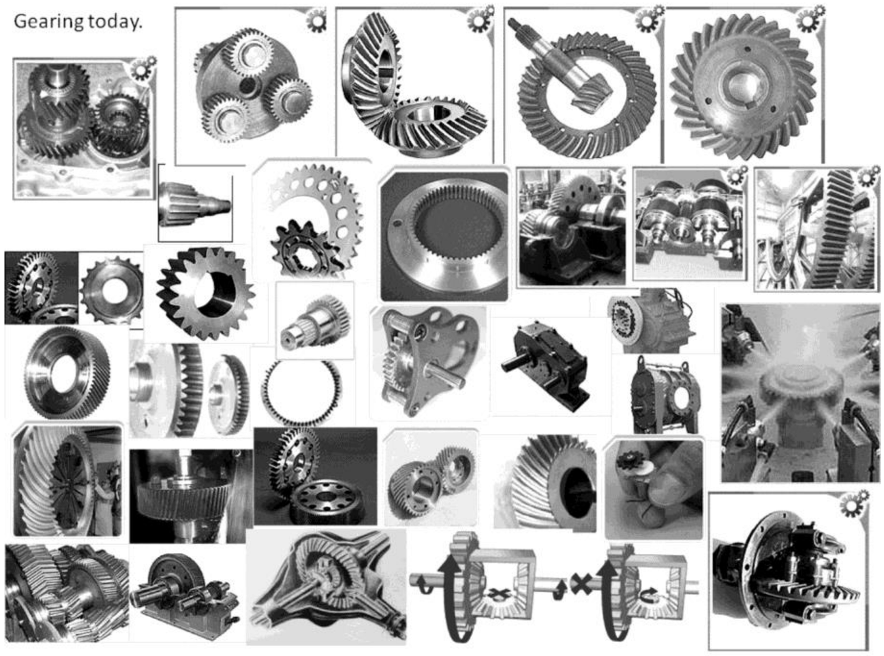

Fig. 6 Gear today

2. Determining Gear Efficiency as a Function of the Contact Ratio

One calculates the efficiency of a geared transmission, having in mind the fact that, at one moment, there are several couples of teeth in contact, and not just one [3]. Hence, the initial model incorporates four pairs of teeth in contact (4 couples) concomitantly [7-11].

The first couple of teeth in contact has the contact point i, defined by ray r_{i1}, and pressure angle α_{i1}. The forces which act at this point are: motor force F_{mi}, perpendicular to position vector r_{i1} at i and the force transmitted from wheel 1 to wheel 2 through point i, F_{ti}, parallel to the path of action and with the direction from wheel 1 to wheel 2. The transmitted force is practically a projection of the motor force onto the path of action. The defined velocities are similar to the forces (having in mind the original kinematics, or the precise kinematics adopted). The same parameters are defined for the other three points of contact – j, k and l (Fig. 7). The quantities of interest are: r_{b1} – the base radius of drive wheel 1; ω_1 – the circular velocity of wheel 1; z_1 – the number of teeth of wheel 1; α – the pressure angle, so that α_1 is the pressure angle for wheel 1 and α_0 is the normal pressure angle on the pitch circle.

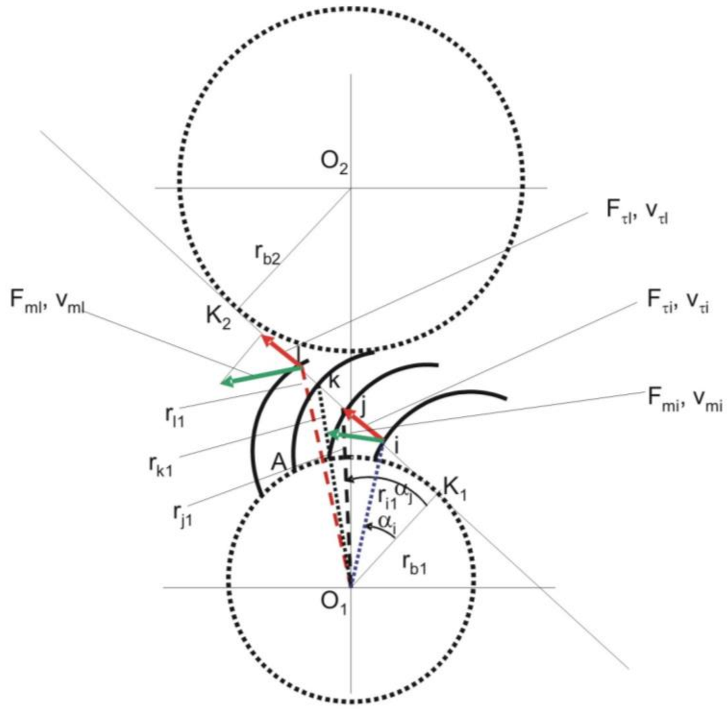

Fig. 7 Four pairs of teeth in contact concomitantly

As a starting point, we write the following relations between the velocities as [8-11]:

$$\begin{aligned} v_{\tau i} &= v_{mi} \cdot \cos\alpha_i = r_i \cdot \omega_1 \cdot \cos\alpha_i = r_{b1} \cdot \omega_1 \\ v_{\tau j} &= v_{mj} \cdot \cos\alpha_j = r_j \cdot \omega_1 \cdot \cos\alpha_j = r_{b1} \cdot \omega_1 \\ v_{\tau k} &= v_{mk} \cdot \cos\alpha_k = r_k \cdot \omega_1 \cdot \cos\alpha_k = r_{b1} \cdot \omega_1 \\ v_{\tau l} &= v_{ml} \cdot \cos\alpha_l = r_l \cdot \omega_1 \cdot \cos\alpha_l = r_{b1} \cdot \omega_1 \end{aligned} \tag{1}$$

From Eqs. (1), one obtains the equality of the tangential velocities (Eq. (2)), and, furthermore, the motor velocities are explicitly obtained (Eq. (3)):

$$v_{\tau i} = v_{\tau j} = v_{\tau k} = v_{\tau l} = r_{b1} \cdot \omega_1 \tag{2}$$

$$v_{mi} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_i}; v_{mj} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_j}; v_{mk} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_k}; v_{ml} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_l} \tag{3}$$

The forces transmitted concomitantly at the four points must be equal [8-11]:

$$F_{\tau i} = F_{\tau j} = F_{\tau k} = F_{\tau l} = F_{\tau} \tag{4}$$

The motor forces are given as:

$$F_{mi}=\frac{F_\tau}{\cos\alpha_i};F_{mj}=\frac{F_\tau}{\cos\alpha_j};F_{mk}=\frac{F_\tau}{\cos\alpha_k};F_{ml}=\frac{F_\tau}{\cos\alpha_l} \tag{5}$$

The momentary efficiency can be written in the following form:

$$\begin{aligned}\eta_i&=\frac{P_u}{P_c}=\frac{P_\tau}{P_m}=\frac{F_{\tau i}\cdot v_{\tau i}+F_{\tau j}\cdot v_{\tau j}+F_{\tau k}\cdot v_{\tau k}+F_{\tau l}\cdot v_{\tau l}}{F_{mi}\cdot v_{mi}+F_{mj}\cdot v_{mj}+F_{mk}\cdot v_{mk}+F_{ml}\cdot v_{ml}}=\\&=\frac{4\cdot F_\tau\cdot r_{b1}\cdot\omega_1}{\frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_i}+\frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_j}+\frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_k}+\frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_l}}=\\&=\frac{4}{\frac{1}{\cos^2\alpha_i}+\frac{1}{\cos^2\alpha_j}+\frac{1}{\cos^2\alpha_k}+\frac{1}{\cos^2\alpha_l}}=\\&=\frac{4}{4+tg^2\alpha_i+tg^2\alpha_j+tg^2\alpha_k+tg^2\alpha_l}\end{aligned} \tag{6}$$

Eqs. (7) and (8) are auxiliary relations [7-11]:

$$\begin{cases}K_1i=r_{b1}\cdot tg\alpha_i;K_1j=r_{b1}\cdot tg\alpha_j;\\K_1k=r_{b1}\cdot tg\alpha_k;K_1l=r_{b1}\cdot tg\alpha_l\\K_1j-K_1i=r_{b1}\cdot(tg\alpha_j-tg\alpha_i);\\K_1j-K_1i=r_{b1}\cdot\frac{2\cdot\pi}{z_1}\Rightarrow tg\alpha_j=tg\alpha_i+\frac{2\cdot\pi}{z_1}\\K_1k-K_1i=r_{b1}\cdot(tg\alpha_k-tg\alpha_i);\\K_1k-K_1i=r_{b1}\cdot 2\cdot\frac{2\cdot\pi}{z_1}\Rightarrow tg\alpha_k=tg\alpha_i+2\cdot\frac{2\cdot\pi}{z_1}\\K_1l-K_1i=r_{b1}\cdot(tg\alpha_l-tg\alpha_i);\\K_1l-K_1i=r_{b1}\cdot 3\cdot\frac{2\cdot\pi}{z_1}\Rightarrow tg\alpha_l=tg\alpha_i+3\cdot\frac{2\cdot\pi}{z_1}\end{cases} \tag{7}$$

$$tg\alpha_j=tg\alpha_i\pm\frac{2\cdot\pi}{z_1};tg\alpha_k=tg\alpha_i\pm 2\cdot\frac{2\cdot\pi}{z_1};tg\alpha_l=tg\alpha_i\pm 3\cdot\frac{2\cdot\pi}{z_1} \tag{8}$$

One keeps Eq. (8), with the sign plus (+) for the gearing where drive wheel 1 has external teeth (regardless if it is external or internal gearing), and with the sign (-) for the gearing where drive wheel 1 has internal teeth (the drive wheel has a ring form only for the internal gearing).

The relation of momentary efficiency (Eq. (6)) uses auxiliary Eq. (8) and takes the form of Eq. (9).

In Eq. (9), one starts with Eq. (6) where four pairs are in contact concomitantly, but then one generalizes the expression by replacing number 4 (four pairs) by variable E, which represents the whole number of the contact ratio +1, and after restricting the sum expressions, variable E is replaced by contact ratio ε_{12}, as well.

The mechanical efficiency offers more advantages than the momentary efficiency, and will be calculated approximately, by replacing pressure angle α_1 in Eq. (9) with normal pressure angle α_0, so that Eq. (10) is obtained, where ε_{12} represents the contact ratio of the gearing, and it will be calculated by Eq. (11) for the external gearing, and by Eq. (12) for the internal gearing:

$$\begin{cases} \eta_i = \dfrac{4}{4+tg^2\alpha_i + tg^2\alpha_j + tg^2\alpha_k + tg^2\alpha_l} = \\ = \dfrac{4}{4+tg^2\alpha_i + (tg\alpha_i \pm \frac{2\pi}{z_1})^2 + (tg\alpha_i \pm 2\cdot\frac{2\pi}{z_1})^2 + (tg\alpha_i \pm 3\cdot\frac{2\pi}{z_1})^2} = \\ = \dfrac{4}{4+4\cdot tg^2\alpha_i + \frac{4\pi^2}{z_1^2}\cdot(0^2+1^2+2^2+3^2) \pm 2\cdot tg\alpha_i \cdot \frac{2\pi}{z_1}\cdot(0+1+2+3)} = \\ = \dfrac{1}{1+tg^2\alpha_i + \frac{4\pi^2}{E\cdot z_1^2}\cdot\sum_{i=1}^{E}(i-1)^2 \pm 2\cdot tg\alpha_i\cdot\frac{2\pi}{E\cdot z_1}\cdot\sum_{i=1}^{E}(i-1)} = \\ = \dfrac{1}{1+tg^2\alpha_1 + \frac{4\pi^2}{E\cdot z_1^2}\cdot\frac{E\cdot(E-1)\cdot(2\cdot E-1)}{6} \pm \frac{4\pi\cdot tg\alpha_1}{E\cdot z_1}\cdot\frac{E\cdot(E-1)}{2}} = \\ = \dfrac{1}{1+tg^2\alpha_1 + \frac{2\pi^2\cdot(E-1)\cdot(2E-1)}{3\cdot z_1^2} \pm \frac{2\pi\cdot tg\alpha_1\cdot(E-1)}{z_1}} \\ \\ \eta_i = \dfrac{1}{1+tg^2\alpha_1 + \frac{2\pi^2}{3\cdot z_1^2}\cdot(\varepsilon_{12}-1)\cdot(2\cdot\varepsilon_{12}-1) \pm \frac{2\pi\cdot tg\alpha_1}{z_1}\cdot(\varepsilon_{12}-1)} \end{cases} \tag{9}$$

$$\eta_m = \frac{1}{1+tg^2\alpha_0 + \frac{2\pi^2}{3\cdot z_1^2}\cdot(\varepsilon_{12}-1)\cdot(2\cdot\varepsilon_{12}-1) \pm \frac{2\pi\cdot tg\alpha_0}{z_1}\cdot(\varepsilon_{12}-1)} \tag{10}$$

$$\varepsilon_{12}^{a.e.} = \frac{\sqrt{z_1^2\cdot\sin^2\alpha_0 + 4\cdot z_1 + 4} + \sqrt{z_2^2\cdot\sin^2\alpha_0 + 4\cdot z_2 + 4} - (z_1+z_2)\cdot\sin\alpha_0}{2\cdot\pi\cdot\cos\alpha_0} \tag{11}$$

$$\varepsilon_{12}^{a.i.} = \frac{\sqrt{z_e^2\cdot\sin^2\alpha_0 + 4\cdot z_e + 4} - \sqrt{z_i^2\cdot\sin^2\alpha_0 - 4\cdot z_i + 4} + (z_i - z_e)\cdot\sin\alpha_0}{2\cdot\pi\cdot\cos\alpha_0} \tag{12}$$

The results of the performed calculations for various gear transmission parameters are summarized in Table 1 [8-11].

Table 1 Gear efficiency for different sets of gear transmission parameters

The summarized results								
z_1	α_0 [°]	z_2	ε_{12}^{ae}	η_{12}^{ae}	η_{21}^{ae}	ε_{12}^{ai}	η_{12}^{ai}	η_{21}^{ai}
42	20	126	1.79	**0.844**	**0.871**	1.92	**0.838**	**0.895**
46	19	138	1.87	**0.856**	**0.882**	2.00	**0.850**	**0.905**
52	18	156	1.96	**0.869**	**0.893**	2.09	**0.864**	**0.915**
58	17	174	2.06	**0.880**	**0.904**	2.20	**0.876**	**0.925**
65	16	195	2.17	**0.892**	**0.914**	2.32	**0.887**	**0.933**
74	15	222	2.30	**0.903**	**0.923**	2.46	**0.899**	**0.942**
85	14	255	2.44	**0.914**	**0.933**	2.62	**0.910**	**0.949**
98	13	294	2.62	**0.924**	**0.941**	2.81	**0.920**	**0.956**
115	12	345	2.82	**0.934**	**0.949**	3.02	**0.931**	**0.963**
137	11	411	3.06	**0.943**	**0.957**	3.28	**0.941**	**0.969**
165	10	495	3.35	**0.952**	**0.964**	3.59	**0.950**	**0.974**
204	9	510	3.68	**0.960**	**0.970**	4.02	**0.958**	**0.980**
257	8	514	4.09	**0.968**	**0.975**	4.57	**0.966**	**0.985**
336	7	672	4.66	**0.975**	**0.980**	5.21	**0.973**	**0.989**
457	6	914	5.42	**0.981**	**0.985**	6.06	**0.980**	**0.992**
657	5	1314	6.49	**0.986**	**0.989**	7.26	**0.986**	**0.994**

3. Efficiency of Helical Gears as a Function of the Contact Ratio

In praxis, helical gears are used very often. For helical gears, the calculations show a decrease in yield with increasing tooth inclination angle (β). For angles not exceeding 25°, the efficiency of gears is rather good. However, when the inclination angle exceeds 25°, the gears will suffer a significant drop in yield.

New calculation relationships can be given in Eqs. (13-15):

$$\eta_m = \frac{z_1^2 \cdot \cos^2\beta}{z_1^2 \cdot (tg^2\alpha_0 + \cos^2\beta) + \frac{2}{3}\pi^2 \cdot \cos^4\beta \cdot (\varepsilon-1)\cdot(2\varepsilon-1) \pm 2\pi \cdot tg\alpha_0 \cdot z_1 \cdot \cos^2\beta \cdot (\varepsilon-1)} \tag{13}$$

$$\begin{aligned}\varepsilon^{a.e.} = \frac{1+tg^2\beta}{2\cdot\pi}\cdot\Big\{&\sqrt{[(z_1+2\cdot\cos\beta)\cdot tg\alpha_0]^2+4\cdot\cos^3\beta\cdot(z_1+\cos\beta)}+\\ &+\sqrt{[(z_2+2\cdot\cos\beta)\cdot tg\alpha_0]^2+4\cdot\cos^3\beta\cdot(z_2+\cos\beta)}-(z_1+z_2)\cdot tg\alpha_0\Big\}\end{aligned} \tag{14}$$

$$\begin{aligned}\varepsilon^{a.i.} = \frac{1+tg^2\beta}{2\cdot\pi}\cdot\Big\{&\sqrt{[(z_e+2\cdot\cos\beta)\cdot tg\alpha_0]^2+4\cdot\cos^3\beta\cdot(z_e+\cos\beta)}-\\ &-\sqrt{[(z_i-2\cdot\cos\beta)\cdot tg\alpha_0]^2-4\cdot\cos^3\beta\cdot(z_i-\cos\beta)}-(z_e-z_i)\cdot tg\alpha_0\Big\}\end{aligned} \tag{15}$$

4. Validation

All the presented relationships have been validated by using the "Inventor" software package; a very good compliance is confirmed. Several examples have been computed for both external and internal gearing. The corresponding gear pairs have been drawn automatically by the "Inventor". Figs. 8 and 9 depict the gear pairs together with the main

parameters of the considered gears for external and internal gearing, respectively. For angle α_0 ranging between 10° and 20°, the contact line is perfect. For values greater than 25°, the software confirms that such a design is no longer safe, and for α_0 taking values lower than 10°, the software has no details necessary for verification because it relies on the experimental values that no longer exist for such small values of angle α_0.

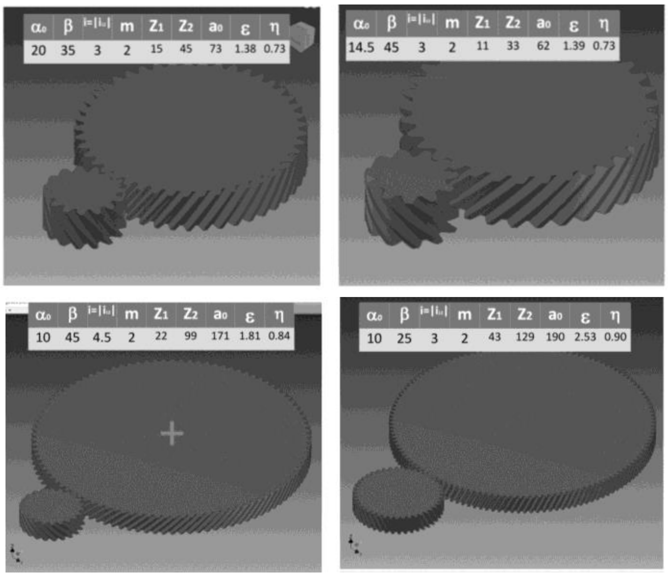

Fig. 8 Validated examples of external gearing

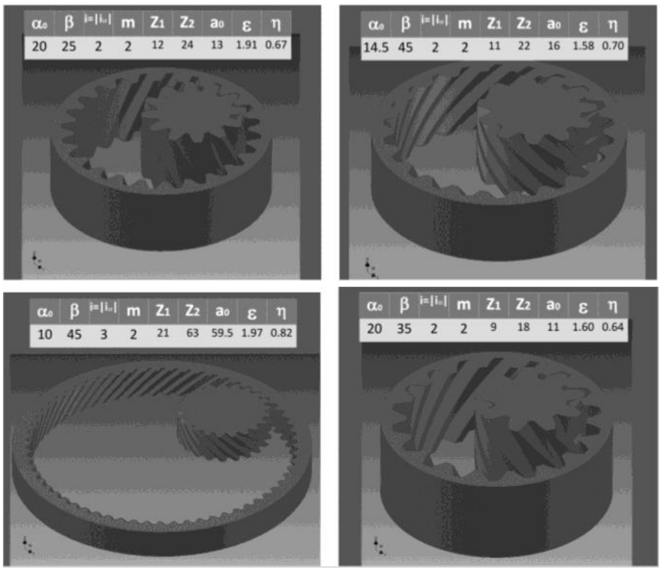

Fig. 9 Validated examples of internal gearing

5. CONCLUSIONS

The best efficiency is obtained with the internal gearing when drive wheel 1 is a ring. The minimum efficiency will be obtained when drive wheel 1 of the internal gearing has external teeth.

For the external gearing, the best efficiency is obtained when the bigger wheel is the drive wheel. With decreasing normal angle α_0, the contact ratio increases and efficiency increases as well.

Efficiency increases too, when the number of teeth of drive wheel 1 (z_1) increases.

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ZUPČANICI VISOKE EFIKASNOSTI

Rad predstavlja originalnu metodu za određivanje efikasnosti zupčastih prenosnika, kao i sila, brzina i snaga u prenosniku. Analizira se način na koji određeni parametri utiču na efikasnost prenosnika. Takođe, ukratko je predstavljena originalna metoda za određivanje efikanosti zupčastih prenosnika u funkciji stepena sprezanja. Pomoću predstavljenih relacija moguće je sprovesti dinamičku sintezu zupčastih prenosnika u cilju postizanja veće efikasnosti mehanizama.

Ključne reči: *zupčanik, zupčasti prenos, stepen sprezanja, dinamička sinteza, efikasnost zupčanika*